International Journal of Applied Mathematics
& Statistical Sciences (IJAMSS)
ISSN(P): 2319–3972; ISSN(E): 2319–3980
Vol. 8, Issue 6, Oct–Nov 2019; 1–10

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations
IASET

SOME PROPERTIES OF CONCURRENT VECTOR FIELDS IN A HYPERSURFACE OF A FINSLER SPACE

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ABSTRACT

Concurrent vector fields in a Finsler space were first of all defined and studied in 1950 by Tachibana [8]. Concurrent vector fields were later on studied by Matsumoto and Eguchi [2] and other. In 2004, Rastogi and Dwivedi [5], while investigating the existence of concurrent vector fields found that the earlier definition of concurrent vector fields in a Finsler space was not suitable and hence, they gave a new definition of concurrent vector fields as follows:

Definition 1

A vector field $X^i(x)$ in a Finsler space F^n in called a concurrent vector field if it satisfies i) $X^i A_{ijk} = \varphi h_{jk}$, ii) $X^i{}_{\mid j} = -\delta^i_j$, where φ is a non-zero arbitrary scalar function of x and y, $A_{ijk} = L C_{ijk}$.

The purpose of the present paper is to investigate the properties of concurrent vector fields by Lie-derivative in a Finsler space F^n. We have also studied some properties of concurrent vector fields in a hypersurface of a Finsler space following an earlier study by Rastogi [6].

KEYWORDS: *Finsler Space, Properties of Concurrent Vector Fields*

Article History

Received: ***27 May 2019*** **|** ***Revised:*** ***08 Oct 2019*** **|** ***Accepted:*** ***19 Oct 2019***

INTRODCUTION

Let F^n be an n=dimensional Finsler space with metric function L(x,y), metric tensor $g_{ij}(x,y)$, angular metric tensor h_{ij} and torsion tensor $A_{ijk} = L C_{ijk}$ Rund [7]. The h- and v-covariant derivatives of a vector field X_i are defined as Matsumoto [4]:

a. $X_{i\mid j} = \eth_j X_i - N^r_j \Delta_r X_i - X_r F^r_{ij}$, (1.1)

b. $X_i \Vert_j = \Delta_j X_i - X_r C^r_{ij}$, (1.1)

Where $N^r_j = F^r_{oj}$, $\eth_j$ and Δ_j respectively denote the partial differentiation with respect to X^j and y^j, such that an index 0 means contraction by unit vector $l^i = y^i L^{-1}$.

The three curvature tensors in a Finsler space are given as follows Matsumoto [4]:

$$R^i_{hjk} = \zeta_{(k,j)}\{\delta_k F^i_{hj} F^i_{rk}\} + C^i_{hr} R^r_{jk}, \quad (1.2)$$

$P_{hijk} = \zeta_{(h,i)} \{ C_{jik}\|_h + C^r_{hj} P_{rik}\}$ (1.3)

and

$S_{hijk} = \zeta_{((k,j)} \{C^r_{hk} C_{ri\,j}\}$, (1.4)

Where $\zeta_{(k,j)}$ denotes interchange of indices k and j and subtraction and $\delta_k = ð_k – N^r_k \Delta_r$.

Let F^{n-1}, be a hypersurface of a Finsler space F^n given by $x^i = x^i (u^{ά})$ and let $B^i_{ά} = ðu^{ά}$, such that $y^i = B^i_{ά}(u) v^{ά}$, where $v^{ά}$ is the element of support of F^{n-1} at $u^{ά}$. Furthermore, the metric and C-tensors of F^{n-1} can be expressed as [3].

$g_{άβ} = g_{ij} B^i_{ά}{}^j_β,\ C_{αβγ} = C_{ijk} B^i_α{}^j_β{}^k_γ$ (1.5)

At each point $u^α$ of F^{n-1}, a unit normal vector $N^i (u,v)$ is defined such that

$g_{ij}(x(u),y(u,v)\ B^i_α N^j = 0,\ g_{ij}(x(u),y(u,v))N^i N^j = 1$ (1.6)

If $B^α_i$ denotes inverse projection factor of $B^i_α$, then we have $B^α_i = g^{α β} g_{ij} B^j_β$, such that

$B^i_α B^β_i = \delta^β_α\ B^α_i N^i = 0,\ B^i_α N_i = 0,\ B^i_α B^α_j = \delta^i_j – N^i N_j$ (1.7)

The induced connection parameter of Cartan connection C, satisfies [3]

$F^α_{βγ} = B^α_i(B^i_{βγ} + F^i_{jk} B^j_β{}^k_γ) + M^α_β H_γ,\ N^α_β = B^α_i(B^i_{0β} + F^i_{0\,j} B^j_β)$, (1.8)

$C^α_{βγ} = B^α_i C^i_{jk} B^j_β{}^k_γ$

Where

$M_{βγ} = N_i C^i_{jk} B^j_β{}^k_γ,\ H_β = N_i(B^i_{0β} + F^i_{0j} B^j_β),\ B^i_{βγ} = ð_γ B^i_β,\ B^i_{0β} = B^i_{α β} V^α$ (1.9)

Further, the second fundamental tensors are given by [3] and satisfy

$H_{βγ} = N_i (B^i_{βγ} + F^i_{jk} B^j_β{}^k_γ) + M_β H_γ,\ M_β = N_i C^i_{j\,k} B^j_β N^k$. (1.10)

$B^i_α\|_β = H_{αβ} N^i,\ B^i_α\|_β = M_{α β} N^i,\ N^i\|_β = - H^α_β B^i_α,\ N^i\|_β = - M^α_β B^i_α$ (1.11)

Concurrent Vector Fields in F^n

Let X_i be an arbitrary covariant vector field, then for the infinitesimal transformation of the type

$x^i = x^i + v^i(x)\, dι$, (2.1)

The Lie- derivative of the vector field X_i can be expressed as follows Rund [7]:

$£X_i = X_{i\,|\,k}V^k + V^r\|_i X_r + (\Delta_h X_i)\,(v^h\|_k)y^k$ (2.2)

Let X_i be a concurrent vector field in F^n, then from equation (.2), we can obtain

$£X_i = -v_i + v^r\|_I X_r$ (2.3)

If we assume that $£X_i = 0$, equation (2.3) gives $v^r\|_I X_r = v_i$. Conversely, if $v^r \| X_r = v_i$ and X_i is a concurrent vector field in F^n, $£X_i = 0$. Hence we have:

Theorem 2.1: The Lie=derivative of a concurrent vector field $X_{i(x)}$ in F^n vanishes if and only if v_i satisfies equation (2.1) and $v^r\|_I X_r = v_i$.

Remarks 1: By taking h-covariant derivative of the product ($X_r V^r$) we can easily obtain that this product is h-covariantly constant.

By taking v-covariant differentiation of the product ($X_r V^r$), we can easily obtain that this product is also v-covariantly constant.

From equation $v^r\|_I X_r = v_i$, we can obtain $v^r\|_i + v_i\|_j$ which gives $X_r (v^r\|_{i j} - v^r\|_{j I}) = 0$. Substituting the value of ($v^r\|_{i j} - v^r\|_{j i}$), we can obtain on simplification

$$X_r v^m R^r{}_{mij} - \varphi R^m{}_{ij} (v_m - v^p l_m) = 0 \tag{2.4}$$

Hence we have:

Theorem 2.2: The sufficient condition for the vector field v^k to satisfy the equation (2.4) is given by vanishing of Lie-derivative of the vector field X_i.

Let v^h be a concurrent vector field in F^n and let X_i be a function of x alone, then equation (2.2) implies $£X_i = X_i\|_k v^k - X_i$. Hence we have:

Theorem 2.3: If v^h is a concurrent vector field in F^n and X_i is a function of x alone, the Lie-drivative of the vector field X_i will vanish if and only if $X_i\|_k v^k = X_{i.}$

Theorem 2.4: If v^h is a concurrent vector field in F^n and X_i is a function of x alone such that its Lie-derivative vanishes, then the vector field X_i satisfies $X_p (R^p{}_{ikm} - C^p{}_{ir} R^r{}_{km}) = 0$.

If in equation (2.1), vector field v^h is replaced by X^h, equation (2.2) leads to $£ X_i = (X_{i\,|\,k} + X_k\|_i) X^k$. In addition to this if X_i is a concurrent vector field, we can obtain $£X_i = -2 X_i$. Hence we have:

Theorem 2.5: If $x^i = x^i + X^i(x)\, d\iota$, is the infinitesimal transformation and X_i is a concurrent vector field them its Lie-derivative satisfies $£X_i = -2 X_i$.

From equation $£X_i = -2 X_i$, we can easily obtain $(£X_i)\|_j = £(X_i\|_j)$,

Hence we have:

Corollary 1: A concurrent vector field X_i satisfying infinitesimal transformation $x^i = x^i + X^i(x)\, d\iota$, also satisfies $(£X_i)\|_j = £(X_i\|_j)$.

Taking v=covariant derivative of $v^r\|_i X_r = v_i$,, we can get

$$\zeta_{(i,j)}\{v^r\|_I\|_j X_r - v^r\|_i L^{-1} \varphi h_{rj}\} = 0 \tag{2.5}$$

Which on simplification leads to?

$$\zeta_{(i,j)} \{L^{-1} \varphi(v^r\|_i h_{rj} - v_{j\,|\,i}) + v^m X_r (C^r{}_{mj\,|\,i} - p^r{}_{mij}\} = 0. \tag{2.6}$$

In a Finsler space F^n with vanishing second curvature tensor Kawaguchi [1], equation (2.6) leads to

$$v_i - v^m l_i l_m + L (\varphi\|_o)^{-1} \varphi l_r v^r\|_j h^j{}_i = 0. \tag{2.7}$$

Hence we have:

Theorem 2.6: If the Lie-derivative of the concurrent vector field X_r vanishes in the Finsler space F^n with vanishing second curvature tensor P_{kjih}, the vector v_i satisfies (2.7).

It is known that $\Delta_m (£X_i) - £ (\Delta_m X_i) = v^k (F^h{}_{mk} \Delta_h X_i - X_h \Delta_m F^h{}_{ik})$, therefore for a concurrent vector field X_i, we can establish:

Theorem 2.7: A vector field $X_i(x)$ with infinitesimal transformation $x^i = x^i + v^i (x)\, d\iota$, satisfies $\Delta_m (£X_i) = - v^k X_h \Delta_m F^h{}_{ik}$.

It is known that $(£X_i)\|_m = C^r{}_{im}\, \varphi X_r - v_r X_h \Delta_m F^h{}_i$ and $£(X_i \|_m) = -£ (X_r C^r{}_{im})$, therefore, we can also establish.

Theorem 2.8: A concurrent vector field X_i with infinitesimal transformation $x^i = X^i + v^i(x)\, d\iota$, satisfies $(£X_i)\|_m - £(X_i \|_m) = (£X_r)C^r{}_{im} - v^r X_h \Delta_m F^h{}_{ir} + H_{im} £\varphi - \varphi £ h_{im.}$

Lie–Transformation in F^n

Definition 3.1: Let $X_i(x)$ be a covariant vector field in F^n, which is transformed to another vector field X_i, then the transformation given by

$$X_i = X_i + £X_i \tag{3.1}$$

Shall be called Lie- transformation of a vector field.

Since we know that $£(X_i g^{ij}) = £X^i$, therefore on substituting the value of Lie-derivatives of X_i and g^{ij}, we get on simplification $X_i C^i{}_{jr} v^r{}_{|k} y^k = 0$, which for a concurrent vector field gives

Theorem 3.1: If $X_i(x)$ is a concurrent vector field in a Finsler space F^n, the vector field v satisfies $v_{j|o} = v^r{}_{|0} l_j l_r$.

It is known that $(\Delta_j \delta_i - \delta_i \Delta_j)X_p = -(\Delta_j N^k{}_i) \Delta_k X_p$ and since $\Delta_k X_p = 0$, therefore from equation (3.1) we can obtain

$$(\Delta_j \delta_i \Delta_j)(X_p - £ X_p) = 0 \tag{3.2}$$

Hence we have:

Theorem 3.2: A vector field $X_i(x)$, satisfying Lie-transformation also satisfies equation Taking h-covariant derivative of equation (3.1), we can obtain

$$X_i\|_j = X_i \|_j + X_i \|_r \|_j v^r + X_i \|_r + v^r\|_j + v^r\|_i \|_j X_r + v^r \|_i X_r\|_j + (\Delta_h X_i) \|_j (v^h \|_k) y^k + (\Delta_h X_i) v^h\|_k \|_j y^k \tag{3.3}$$

If we assume that vector field X_i is a concurrent vector field in F^n, equation (3.3) on simplification gives $\zeta_{(i,j)}(X_i \|_j - v^r \|_i \|_j X_r) = 0$, which on further simplification leads to

$$\zeta_{(i,j)}(X_i \|_j) - \{X_r v^m R^r{}_{mij} - \varphi R^m{}_{ij}(v_m - v^p l_p l_m)\} = 0 \tag{3.4}$$

If $\zeta_{(i,j)}(X_i \|_j) = 0$, equation (3.4) gives (2.4), Hence we have:

Theorem 3.3: The necessary and sufficient condition for the h-covariant derivative of Lie-transformation of a concurrent vector field X_i to be symmetric is the vector field X_i satisfies equation (2.4).

Taking v-covariant derivative of equation (3.1) and assuming X_i to be a concurrent vector field, we can obtain

$$X_r(v^m p^r{}_{mij} - v^r \|_m C^m{}_{ij} - v^r \|_m p^m{}_{ij} + v^m\|_i C^r{}_{mi} - v^m \|_j C^r{}_{mi}) = 0 \tag{3.5}$$

which yields

Theorem 3.4: A concurrent vector field X_i, satisfying Lie-transformation also satisfies equation (3.5).

Taking h-covariant derivative of $X_i \|_j$ and h-covariant derivative of $X_i \|_k$ and simplifying the subtraction, after some lengthy calculation we obtain

$$2C_{ijk} - v^r \|_i C^r_{kj} - v_k \|_I \|_j = X_r \{P^r_{ijk} + C^r_{ik} \|_j + C^r_{mk} v^m \|_i \|_j - P^h_{jk} C^r_{ih} + v^m \|_I (P^r_{mjk} + C^m_{hk} \|_j)\} \quad (3.6)$$

Hence we have:

Theorem 3.6: In a Finsler space F^n, if a concurrent vector field X_i satisfies equation (3.1), curvature tensor P^r_{ijk} satisfies equation (3.6).

Taking v-covariant derivative of $X_i \|_j$, finding $X_i \|_j \|_k \|_j - X_i \|_k \|_j$ and taking cyclic summation in i,j,k, we obtain on simplification

$$\Sigma_{(i,j,k)}\{v^m(\Delta_j F^r_{im} - \Delta_i F^r_{jm})\} = 0. \quad (3.7)$$

Hence we have:

Theorem 3.7: In a Finsler space F^n, if a concurrent vector field X_i satisfies equation (3.1), the connection parameter F^r_{ij} satisfies equation (3.7).

Concurent Vector Fields in F^{n-1}

Let X_i be a concurrent vector field in F^n and let a point of F^{n-1}, it is written as

$$X_{i(x)} = X_\alpha B^\alpha_i + \mu N_i \quad (4.1)$$

Where $X_\alpha = X_i B^i_\alpha \mu = X_i Ni$. It is known that $ð/ð_v^\beta = B^j_\beta (ð/ð y^i)$, $(ð/ðv^\beta) B^i_\alpha = 0$, therefore from the fact that X_i is a function of x and equation (4.1), we can obtain that $(ð/ðv^\beta) X_\alpha = 0$. Hence X_α is a function of coordinate u only. We know that $X_{\alpha \mid \beta} = X_i \|_\beta B^i_\alpha + X_i B^i_{\alpha \mid \beta}$, therefore, substituting from equation (1.11), we get $X_\alpha \|_\beta = X_i \|_\beta B^i_\alpha + \mu H_{\alpha\beta}$. Further substituting from $X_i \|_\beta = X_i \|_j N^j H_\beta$, we can obtain on simplification.

$$X_\alpha \|_\beta = -g_{\alpha\beta} + \mu H_{\alpha\beta} \quad (4.2)$$

Hence we have:

Theorem 4.1: The necessary and sufficient condition for the component $X_{\alpha(u)}$ in F^{n-1}o f concurrent vector field $X_i(x)$ in F^n to be the component of a concurrent vector field in F^{n-1}, is that either X_i is tangential to the hyper-surface F^{n-1} or $H_{\alpha\beta}$, the h-fundamental tensor of F^{n-1} vanishes.

If $H_{\alpha\beta} = 0$, we can obtain

$$N_i (B^i_{\beta\gamma} + F^i_{jk} B^j_\beta{}^k_\gamma) = -M_\beta H_\gamma \quad (4.3)$$

Thus we have:

Corollary 2: If the vector X_i is not tangential to the hyper-surface F^{n-1}, vectors $X_i B^i_\alpha \|_\beta$, on simplification we can obtain with the help of equations (1.5), (1.7) and (4.1)

$$X_\alpha C^\alpha_{\beta\gamma} = L^{-1} \varphi h_{\beta\gamma} - \mu M_{\beta\gamma} \quad (4.4)$$

It can also be observed that if X_α is a concurrent vector field in F^{n-1}, then

$$X_\alpha C^\alpha_{\beta\gamma} = (L)^{-1} \psi h_{\beta\gamma} \quad (4.5)$$

Where L and ψ are terms defined in F^{n-1}, similar to L and φ of F^n.

Comparing equations (4.3) and (4.4), we can observe that

$$(L^{-1}\varphi-(l)^{-1}\psi) h_{\beta\gamma} = \mu M_{\beta\gamma} \quad (4.6)$$

Hence we have:

Theorem 4.2: The necessary and sufficient condition for X_i and X_α, to b concurrent n F^n and F^{n-1} respectively is that v-fundamental tensor is proportional to angular metric tensor in F^{n-1}.

It is known that for a hyper-plane of third kind F^{n-1}, Matsumoto [3], $H_{\alpha\beta}$ and $M_{\alpha\beta}$ vanish, which leads to

Theorem 4.3: If X_i is a concurrent vector field in F^n, X_α, will be a concurrent vector field in a hyper-plane of third kind.

Differentiating equation (4.2) covariantly with respect to u^γ, w get on simplification

$$X_\alpha \|_\beta \|_\gamma - X_\alpha \|_\gamma \|_\beta = \mu(H_{\alpha\beta} \|_\gamma - H_{\alpha\gamma} \|_\beta) + \mu \|_\gamma H_{\alpha\beta} - \mu \|_\beta H_{\alpha\beta} \quad (4.7)$$

Substituting the value of left hand side in (4.7), we get

$$\mu (H_{\alpha\beta}\|_\gamma - H_{\alpha\gamma} \|_\beta) + \mu \|_\gamma H_{\alpha\beta} - \mu \|_\beta H_{\alpha\beta} + X_\delta R^\delta{}_{\alpha\beta\gamma} - X_\theta C^\theta{}_{\alpha\delta} R^\delta{}_{\beta\gamma} = 0$$

Which for a concurrent vector field X_α in F^{n-1} leads to?

$$\mu(H_{\alpha\beta} \|_\gamma - H_{\alpha\gamma} \|_\beta) + \mu \|_\gamma H_{\alpha\beta} - \mu_{|\beta} H_{\alpha\beta} = 0 \quad (4.8)$$

Conversely, if equation (4.8) is satisfied, equation (4.7) leads to

$$X_\delta R^\delta{}_{\alpha\beta\gamma} + L^{-1} \psi h_{\alpha\delta} R^\delta{}_{\beta\gamma} = 0 \quad (4.9)$$

Hence we have:

Theorem 4.4: If X_α is a concurrent vector field in F^{n-1}, it is necessary condition that second fundamental tensor $H_{\alpha\beta}$ satisfies (4.8), conversely, if equation (4.8) is satisfied, it is sufficient that concurrent vector field X_α satisfies (4.9).

Since $X_\alpha \|_\beta = X_i \|_j B^i{}_\alpha B^j{}_\beta + X_i B^i{}_\alpha \|_\beta$, therefore on simplification we get

(4.10) $X_\alpha \|_\beta = - L^{-1} \varphi h_{\alpha\beta} + \mu M_{\alpha\beta}$,

Which leads to?

Theorem 4.5: If X_i is a concurrent vector field in F^n, X_α will be concurrent vector field in F^{n-1}, if and only if $L^{-1} \varphi = (L)^{-1} \psi$ and either $\mu = 0$, i.e, X_i is tangential to the hyper-surface F^{n-1} or $M_{\alpha\beta} = 0$.

From equation (4.10), we can obtain $X_\alpha \|_\beta \|_\gamma - X_\alpha \|_{\gamma\beta} = L^{-1} (\psi \|_\beta h_{\alpha\gamma} - \psi \|_\gamma h_{\alpha\beta})$, which on simplification leads to

$$L^{-1} (\psi \|_\beta h_{\alpha\gamma} - \psi \|_\gamma h_{\alpha\beta}) + X_\theta S^\theta{}_{\alpha\beta\gamma} = 0 \quad (4.11)$$

Hence we have:

Theorem 4.6: If X_i is a concurrent vector field in F^n, X_α will be a concurrent vector field in F^{n-1}, if and only if curvature tensor $S^\theta{}_{\alpha\beta\gamma}$ satisfies equation (4.11).

If X_α is a concurrent vector field in F^{n-1}, then from $X_\alpha \| _\beta = -g_{\alpha\beta}$, we can obtain $X_\alpha \| _\beta \| _\gamma - X_\alpha \| _\gamma \| _\beta = X_\theta \| _\beta C^\theta{}_{\alpha\gamma} + X_\theta C^\theta{}_{\alpha\gamma} \| _\beta$, which on simplification leads to

$$X_\theta (P^\theta{}_{\alpha\beta\gamma} + C^\theta{}_{\alpha\gamma} \| _\beta - C^\theta{}_{\alpha\varphi} P^\varphi{}_{\beta\gamma}) = 2\, C_{\alpha\beta\gamma} \tag{4.12}$$

Hence we have:

Theorem 4.7: If X_θ is a concurrent vector field in F^{n-1}, curvature tensor $P^\theta{}_{\alpha\beta\gamma}$ satisfies equation (4.12).

Lie-Derivative in F^{n-1}

Taking Lie-derivative of the relation $X_\alpha = X_i B^i{}_\alpha$ and using $u^\alpha = u^\alpha + w^\alpha(u)d\iota$, we can obtain

$$X_\alpha \| _\gamma v^\gamma \| _\alpha X_\gamma = (X_i \| _j v^j + v^j \| _I X_j) B^i{}_\alpha \tag{5.1}$$

Which for concurrent vector fields X_i and X_α leads to?

$$(-v_i + v^k \| _j X_k) B^i{}_\alpha = - w_\alpha + w^\gamma \| _\alpha X_\gamma \tag{5.2}$$

Hence we have:

Theorem 5.1: If X_i and X_α are respectively concurrent vector fields in F^n and F^{n-1}, they satisfy equation (5.2).

Since $X_i N^i = \mu$, therefore for a concurrent vector field X_i we can easily obtain

$$\pounds\, \mu = X_i\, \pounds\, N^i (-v_i + v^k \| _I X_k), \tag{5.3}$$

which implies

Theorem 5.2: If X_i is a concurrent vector field satisfying $X_i N^i = \mu$, the Lie-derivative of the scalar μ is given by equation (5.3).

If in particular, we replace vector field v by X, equation (5.2), on simplification gives

$$X_\alpha = (1/2)(w_\alpha - w^\gamma \| _\alpha X_\gamma), \tag{5.4}$$

Which implies?

Corollary 3: If X_i and X_α are respectively concurrent vector fields in F^n and F^{n-1} and satisfy coordinate transformations $x^i = x^i + X^i(x)\, d\iota$ and $u^\alpha + w^\alpha(u)d\iota$, then the vector field X_α satisfies (5.4).

Replacing v_i by X_i n equation (5.3), we get

Corollary 4: A concurrent vector field X_i in F^n satisfying coordinate transformation $x^i = x^i + X^I(x)\, d\iota$, also satisfies $\pounds\, \mu = X_i\, \pounds\, N^i - 2\,\mu$.

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Ashwini Kumar Dwivedi completed his Bachelor of Science (1992), Master of Science (1994) and Ph.D. Degree (2003) from Dr. R.M.L Avadh University, Faizabad (Ayodhya), U.P. India in the field of Mathematics with a specialization in Differential Geometry. Dr. Dwivedi has extensive experience in the field of Academics with an eminent career spanning over 20 years. He has held various positions of importance and shouldered various responsibilities like ensuring fair conduct of examinations and monitoring academic activities. He is also a part of the Academic Committee of his organization which regularly updates and regulates the course curriculum. Displaying keen academic acumen, Dr. Dwivedi has published around 20 papers in journals and conferences, of national and international repute. Profoundly known in his field as well conversant with the academic norms and regulations, he offers guidance and counseling to his peer members in matters related to academic administration. Currently he is serving as Dean (Academics) in Central Institute of Plastics Engineering and Technology, Lucknow, where he oversees all academic activities. .: